Written Exam, Summer school, Economics summer 2011

Micro 3

Final Exam

July 25th, 2011

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.
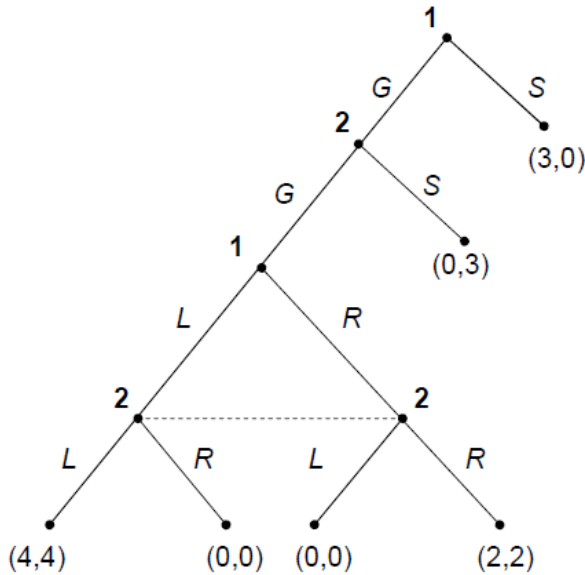
Problem 1

Consider the following normal form game:

	A_2	B_2	C_2	D_2
A_1	4,3	5,0	3,1	2,0
B_1	1,1	2,5	1,4	4,0
C_1	2,2	4,3	5,5	6,1

(a) Analyze the game by iterated elimination of strictly dominated strategies. Describe each step. Which strategies survive?

Problem 2

Consider the game given by the following game tree:

(a) How many subgames are there in the game (excluding the game itself)?

(b) Describe the set of possible strategies for each player.

(c) Write down the normal form ("matrix form") of the game and find all pure strategy Nash equilibria.

(d) Find all the pure strategy subgame-perfect Nash equilibira (SPE) of the game

Problem 3

In a duopoly each of the two firms produces a homogenous good for which the inverse demand is given by:

$$p = 7 - Q + A_1 + A_2,$$

where Q is the total quantity sold, p is the price and $A_i \geq 0$ is firm i's advertising effort, $i = 1, 2$. The cost of making an advertising effort of A_i is $(A_i)^2$. The production of firm i, $q_i \geq 0$, is produced at zero costs, and $Q = q_1 + q_2$. The game between the firms has two stages. First each firm chooses its advertising effort *without* knowing the effort of the other firm. When both firms have chosen their efforts, the total effort is revealed. Then each firm chooses its produced quantity *without* knowing the other firm's production. Then a market clearing price is formed, and each firm receives a profit, which is the firm's payoff in the game.

(a) Sketch an extensive form of the game. Find the payoffs π_1 and π_2 to the firms as functions of A_1, A_2, q_1 and q_2.

(b) Find for given advertising efforts A_1 and A_2 the Nash equilibrium in the subgame starting when firms have chosen A_1 and A_2. Find, as function of A_1 and A_2, the quantities q_1 and q_2 produced and the market price.

(c) Find the subgame perfect equilibrium of the entire game. State the market price, and the production, advertising effort, and profit of each firm in this equilibrium.

Problem 4

Consider the following game in strategic form, where a is some real number.

	L	R
U	5 , 1	4 , a
D	2 , 1	5 , 0

(a) Find for all values of a all (pure and mixed) Nash-equilibria of the game.

Assume that player 2 knows the value of a, whereas player 1 just knows that $a = 3$ with probability 0.5 and $a = -3$ with probability 0.5.

(b) Explain that this situation can be modeled as a Bayesian game, i.e., describe the players, actions, types, prior and utility functions in the Bayesian game modeling this situation.

(c) Find the Bayesian Nash equilibrium of this Bayesian game.

Problem 5

Three firms, A, B and C, use a similar technology, even though they produce different products and are not competing in the product market. They negotiate to pool some of their research and development resources (laboratories and engineers) in a development project that can improve the production efficiency in each of the firms. They expect that after having covered the development costs, they can increase their joint profit by 100 from undertaking the project.

Each firm can also attempt the project alone or with just one of the other firms, but that is expected to be relatively more costly. In particular firm A expect that it can earn a profit 40 by doing the project alone, where as firms B and C being somewhat smaller and lacking a core facility that firm A holds, can each only earn 20 from undertaking the project alone, and 50 in total if they work together. If A undertakes the project with either B or C, the joint profit of the pair is expected to be 70.

(a) Think of this situation as a coalitional game with transferable payoff and write down the value of all coalitions.

(b) Find the core of this game.

(c) Discuss briefly how the core is relevant for predicting the outcome of the negotiations among the firms.